Stopping of directed energetic electrons in high-temperature hydrogenic plasmas

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From fundamental principles, the interaction of directed energetic electrons with a high-temperature hydrogenic plasma is analytically modeled. The randomizing effect of scattering off both plasma ions and electrons is treated from a unified point of view. For electron energies less than 3 MeV, electron scattering is equally important. The net effect of multiple scattering is to reduce the penetration from 0.54 to 0.41 $\mathrm{g/cm^2}$ for 1 MeV electrons in a 300 $\mathrm{g/cm^3}$ plasma at 5 keV. These considerations are relevant to "fast ignition" and to fuel preheat for inertial confinement fusion.

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A basic problem in plasma physics is the interaction and energy loss of energetic charged particles in plasmas [1–4]. This problem has traditionally focused on ions (i.e., protons, α particles, etc.), either in the context of heating and/or ignition in, for example, inertially confined fusion (ICF) [3–6]; or in the use of these particles for diagnosing implosion dynamics [7]. More recently, prompted in part by the concept of fast ignition for ICF [8], workers have begun considering energy deposition from relativistic fast electrons in deuterium-tritium (DT) plasmas [8–13]. Tabak *et al.* [8] used, for example, the energy deposition of Berger and Seltzer [14] that is based on the continuous slowing down of electrons in cold matter. This treatment, though quite similar to electron slowing in plasmas, does not include the effects of scattering. Deutsch *et al.* [9] addressed this issue by considering the effects of scattering off the background ions [16,17]; they ignored scattering due to background electrons.

In another important context in ICF, workers addressed the issue of fuel preheat due to energetic electrons ($\sim$50–300 keV) [5,18,19], the consequence of which is to elevate the fuel adiabat to levels that would prohibit ignition. Herein we show that scattering effects could be significant for quantitative evaluations of preheat.

The starting point for these calculations is the relativistic elastic differential cross sections for electrons scattering off fully ionized ions of charge Z [20–22], and off the neutralizing bath of electrons [23,21,24], which are approximated as

$$\left(\frac{d\sigma}{d\Omega}\right)^{ei} \approx \frac{Z^2}{4}\left(\frac{r_0}{\gamma\beta^2}\right)^2 \frac{1}{\sin^4(\theta/2)}, \tag{1}$$

$$Z\left(\frac{d\sigma}{d\Omega}\right)^{ee} \approx Z\frac{(\gamma+1)^2}{(2^{\sqrt{(\gamma+1)/2}})^4}\left(\frac{r_0}{\gamma\beta^2}\right)^2 \frac{1}{\sin^4(\theta/2)}, \tag{2}$$

where $\beta = v/c$ and $\gamma = (1-\beta^2)^{-1/2}$; $r_0 = e^2/m_0c^2$ is the classical electron radius. The relative importance of electron scattering is implied from the ratio

$$\mathcal{R} = Z\left(\frac{d\sigma}{d\Omega}\right)^{ee} \Bigg/ \left(\frac{d\sigma}{d\Omega}\right)^{ei} \approx \frac{4(\gamma+1)^2}{(2^{\sqrt{(\gamma+1)/2}})^4}\frac{1}{Z}. \tag{3}$$

For a hydrogenic plasma $(Z=1)$ and for $\gamma \lesssim 10$, $\mathcal{R} \sim 1$, indicating that the electron component is equally important. As best we can tell, the electron scattering component has been ignored by workers since it was typically assumed, usually justifiably, that ion scattering dominates. However, this will not be the case for problems discussed herein, for relativistic astrophysical jets [25], or for many of the present high-energy laser plasma experiments [26] for which Z is about 1 and for which $\gamma \lesssim 10$.

To calculate the effects of multiple scattering a diffusion equation is used [27],

$$\frac{\partial f}{\partial s} + \mathbf{v}\cdot\nabla f = n_i \int [f(\mathbf{x},\mathbf{v}',s) - f(\mathbf{x},\mathbf{v},s)]\sigma(|\mathbf{v}-\mathbf{v}'|)d\mathbf{v}', \tag{4}$$

where f is the distribution function of the scattered electrons, n_i is the number density of plasma ions of charge Z, $\mathbf{x}$ is the position where scattering occurs, and $\sigma = \sigma_{ei} + Z\sigma_{ee}$ is the total scattering cross section where $\sigma_{ei} = \int (d\sigma/d\Omega)^{ei} d\Omega$ and $\sigma_{ee} = \int (d\sigma/d\Omega)^{ee} d\Omega$. Equation (4) is solved in a cylindrical coordinates with the assumption that the scattering is azimuthally symmetric. The solution that satisfies the boundary conditions is [27,28]

$$f(\theta,s) = \frac{1}{4\pi}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\exp\left(-\int_0^S \kappa_\ell(s')ds'\right), \tag{5}$$

where $P_\ell(\cos\theta)$ are the Legendre polynomials. Using orthogonality and projecting the $\ell=1$ term,

$$\langle\cos\theta\rangle = \exp\left(-\int_{E_0}^{E} \kappa_1(E)\left(\frac{dE}{ds}\right)^{-1} dE\right), \tag{6}$$

where $\langle\cos\theta\rangle$, a function of the residual electron energy E and the initial energy E_0, is a measure of the mean deflection resulting from multiple scattering [29], and relates dE/ds to dE/dx through

$$\frac{dE}{dx} = \langle\cos\theta\rangle^{-1}\frac{dE}{ds}, \tag{7}$$

where dE/ds is the stopping power along the path while dE/dx is the linear energy stopping power. In the above, $S(E) = \int_0^S ds' = \int_{E_0}^{E}(dE/ds)^{-1}dE$, and

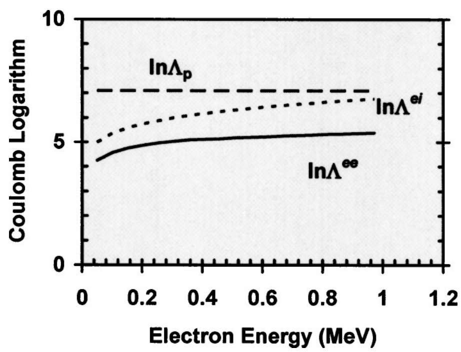

FIG. 1. The Coulomb logarithms for incident 1 MeV electrons interacting with a DT plasma ($\rho=300\ \mathrm{g/cm^3}$, $T_e=5$ keV). For the background plasma the Coulomb logarithm $\ln \Lambda_p$, which is relevant to plasma transport processes (e.g., electrical and thermal conductivity), is about 7.

$$\kappa_1(E) = 2\pi n_i \int_0^\pi \left(\frac{d\sigma}{d\Omega}\right)(1-\cos\theta)\sin\theta\, d\theta, \tag{8}$$

where κ_1 is closely related to the diffusion cross section (or transport cross section) which characterizes the loss of directed electron velocity through scattering [2]. Equations (1) and (2) are substituted into Eq. (8) and, after a standard change of variables, the integrations are taken from $b_{\min}^{ei}$ or $b_{\min}^{ee}$ to λ_D, where λ_D is the Debye length, and $b_{\min}^{ei}$ ($b_{\min}^{ee}$) is the larger of b_q^{ei} (b_q^{ee}) or $b_\perp^{ei}$ ($b_\perp^{ee}$) [30]. b_q^{ei} and b_q^{ee} are approximately the electron de Broglie wavelength, and $b_\perp^{ei} = Zr_0/\gamma\beta^2$ and $b_\perp^{ee} \approx 2(\gamma+1)r_0/[(2^{\sqrt{(\gamma+1)/2}})^2\gamma\beta^2]$ are the impact parameters for 90° scattering of electrons off ions or electrons off electrons. Thus $[\kappa_1(E)=\kappa_1^{ei}(E)+Z\kappa_1^{ee}(E)]$

$$\kappa_1(E) = 4\pi n_i\left(\frac{r_0}{\gamma\beta^2}\right)^2\left[Z^2\ln\Lambda^{ei} + \frac{4(\gamma+1)^2}{(2^{\sqrt{(\gamma+1)/2}})^4}Z\ln\Lambda^{ee}\right], \tag{9}$$

where the arguments of the Coulomb logarithm are $\Lambda^{ei} = \lambda_D/b_{\min}^{ei}$ and $\Lambda^{ee}=\lambda_D/b_{\min}^{ee}$ [30]. As these Coulomb logarithms are used in this and later calculations, they are shown in Fig. 1.

The stopping power contained in Eq. (6) consists of contributions from binary interactions with plasma electrons and from plasma oscillations. The binary contribution is [31]

$$(dE/ds)_b = -n_i Z(\gamma-1)m_0c^2\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \varepsilon(d\sigma/d\varepsilon)d\varepsilon,$$

where the differential energy loss cross section is from Møller [23],

$$\frac{d\sigma}{d\varepsilon} = \frac{2\pi r_0^2}{(\gamma-1)\beta^2}\left(\frac{1}{\varepsilon^2} + \frac{1}{(1-\varepsilon)^2} + \left(\frac{\gamma-1}{\gamma}\right)^2 - \frac{2\gamma-1}{\gamma^2\varepsilon(1-\varepsilon)}\right), \tag{10}$$

and ε is the energy transfer in units of $(\gamma-1)m_0c^2$. The lower integration limit reflects the minimum energy transfer, which occurs when an incident electron interacts with a plasma electron at λ_D, i.e., $\varepsilon_{\min} \approx 2\gamma {r_0}^2/[\lambda_D(\gamma-1)]^2$ (unless $\gamma\to 1$, the limit for which quantum effects need to be included). The upper limit occurs for a head-on collision, for which $\varepsilon_{\max}=0.5$.

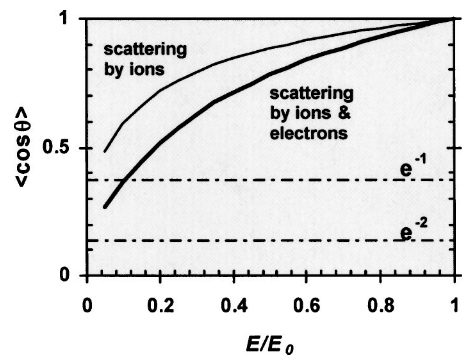

FIG. 2. The mean deflection angle $\langle\cos\theta\rangle$ is plotted against the fraction of the residual energy in a DT plasma for $e\to i$ and for $e\to i+e$ scattering (1 MeV electrons with $\rho=300\ \mathrm{g/cm^3}$, $T_e=5$ keV).

The contribution from plasma oscillations, which reflects the response of the plasma to impact parameters larger than λ_D [30], is

$$(dE/ds)_c = -4\pi r_0^2 m_0c^2 n_i Z\beta^{-2}\ln(1.123\beta/\sqrt{2kT_e/m_0c^2}),$$

where relativistic effects are approximately included. Consequently,

$$\frac{dE}{ds} = -\frac{2\pi r_0^2 m_0c^2 n_i Z}{\beta^2}\left[\ln\left(\frac{(\gamma-1)\lambda_D}{2\sqrt{2\gamma}r_0}\right)^2 + 1 + \frac{1}{8}\left(\frac{\gamma-1}{\gamma}\right)^2 - \left(\frac{2\gamma-1}{\gamma}\right)\ln 2 + \ln\left(\frac{1.123\beta}{\sqrt{2kT_e/m_0c^2}}\right)^2\right]. \tag{11}$$

Utilizing Eq. (11) in Eq. (6), Fig. 2 illustrates the circumstance when the incident electron ($E_0=1$ MeV) continuously changes direction as it loses energy. When $\langle\cos\theta\rangle$ equals one e-folding, $|\theta|\approx 68°$ and $E/E_0\approx 0.1$, at which point the incident electron has lost memory of its initial direction.

Utilizing this result in Eq. (7), Fig. 3 illustrates the enhancement of dE/dx for scattering off ions and for scattering

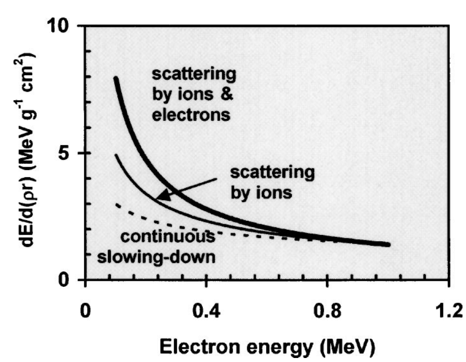

FIG. 3. Stopping power for linear energy transfer and continuous slowing down are plotted as a function of the electron energy for incident 1 MeV electrons in a DT plasma ($\rho=300\ \mathrm{g/cm^3}$, $T_e=5$ keV). Enhancement of dE/dx (solid lines) over dE/ds (dashed line) is a consequence of the effects of multiple scattering.

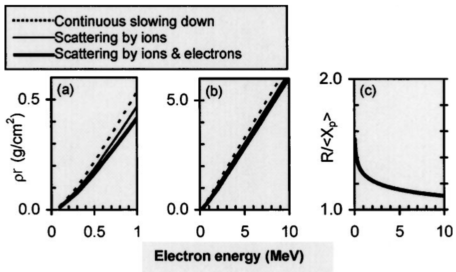

FIG. 4. The range (dashed line) and penetration for 0.1–1 MeV electrons (a) and for 1–10 MeV electrons (b) in a DT plasma ($\rho = 300$ g/cm^3, $T_e = 5$ keV). The penetration is shown for scattering off ions, and for scattering off ions plus electrons. (c) shows the ratio of range to penetration for 0.1–10 MeV electrons. As the initial electron energy decreases, the effects of multiple scattering become more pronounced, and the penetration is further diminished with respect to the range.

off ions plus electrons. This enhancement is further illustrated in Fig. 4 where the corresponding set of curves for the range (R) and the penetration ($\langle X_p \rangle$) are shown for electrons with $E_0 = 0.1$–10 MeV. $R = \int_0^R ds' \approx \int_{E_0}^{\sim kT} (dE/ds)^{-1} dE$, and

$$\langle X_P \rangle \approx \int_{E_0}^{E_1} \langle \cos \theta \rangle \left(\frac{dE}{ds} \right)^{-1} dE, \tag{12}$$

where E_0 is the initial energy; E_1 corresponds to one e-folding of $\langle \cos \theta \rangle$ (see Fig. 2). R is the total path length the electron traverses as it scatters about and eventually thermalizes; $\langle X_p \rangle$ is the average penetration along the *initial* electron trajectory. Contributions from electron and ion scattering are shown in Fig. 4.

Three other points are worth noting: First, the temperature and density dependence are weak, i.e., a factor of 10 reduction in either temperature or density results in only ~10% reduction in the penetration. Second, as the initial electron energy decreases, the effects of scattering become more pronounced [Fig. 4(c)], an effect, very similar in nature, that is also seen in the scattering of energetic electrons in metals [33]. And third, for a given electron energy, scattering effects slightly decrease as the target plasma temperature decreases, i.e., the path of the electron slightly straightens as the target plasma temperature drops. For example, when the target plasma temperature changes from 5.0 to 0.5 keV ($\rho = 300$ g/cm^3), the ratio $R/\langle X_p \rangle$ is reduced by ~5% for 1 MeV electrons.

With the calculation of the penetration as a function of energy loss, the linear energy deposition can be evaluated (Fig. 5). In addition to the differences in total penetration with and without scattering contributions, it is seen that the linear energy transfer increases near the end of its penetration (i.e., an effective Bragg peak), an effect which is seen more weakly with just ion scattering. Such differences may need to be considered in quantitatively modeling the energy deposition of relativistic electrons for fast ignition, and for

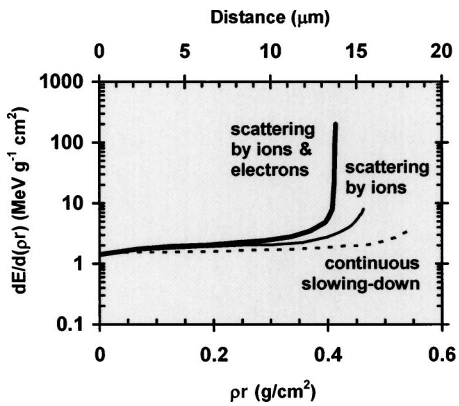

FIG. 5. The stopping power for 1 MeV electrons, plotted as a function of the electron penetration, for a DT plasma with $\rho = 300$ g/cm^3 and $T_e = 5$ keV. The three curves correspond to three different models. As a result of the scattering effects, the energy transfer increases notably near the end of the penetration (i.e., an effective Bragg peak). For these 1 MeV electrons, the effects of scattering reduce the penetration from 0.54 to 0.41 g/cm^2 [32].

critically assessing ignition requirements [34]. It is also interesting, and a consequence of selecting 1 MeV electrons (Figs. 4 and 5), that the effects of scattering reduce the penetration from 0.54 to 0.41 g/cm^2; this latter value is close to the range of 3.5 MeV α particles, 0.3 g/cm^2, which is required for hot-spot ignition in a 10 keV plasma [3–6].

Finally, in order to explore the importance of electron-on-electron multiple scattering in a hydrogenic setting, and as definitive stopping power experiments in plasmas are extremely difficult, we propose that experiments be undertaken in which a monoenergetic electron beam, with energy between 0.1 and 1.0 MeV, scatters off thin layers of either D_2 or H_2 ice, where the thickness of the ice layer is between ~100 and 1000 μm, the appropriate thickness depending on the exact electron energy. Although there are differences in the scattering calculations for cold, condensed hydrogenic matter and a hydrogenic plasma, there is reason to believe that the *relative* importance of the electron-to-electron and the electron-to-ion multiple scattering terms will be approximately the same for both states of matter.

In summary, the energy loss and penetration of energetic electrons into a hydrogenic plasma has been analytically calculated, and the effect of scattering off ions and electrons is treated from a unified point of view. In general scattering enhances the electron linear-energy transfer along the initial electron direction, and reduces the electron penetration. Energy deposition increases near the end of its range. These results should have relevance to "fast ignition" and to fuel preheat in inertial confinement fusion, specifically to energy deposition calculations that critically assess quantitative ignition requirements.

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